STAT C-301
SAMPLING DISTRIBUTION

B. Sc. (H) Statistics Sem III

Unique Paper Code : 32371301

Name of the Paper : Sampling Distributions

1. Find m. g. f. of chi- square distribution and obtain limiting form of χ^2 distribution for large degree of freedom.

2. Find Mode of χ^2 distribution and describe the chi-square probability curve.

3. For a chi=square distribution with n df . Establish recurrence relation between the moments. Hence find β_1 and β_2.

4. Find the p.d.f. of $\chi_{(n)} = \sqrt{\chi^2_{(n)}}$ where $\chi^2_{(n)}$ is a chi-square with n d.f. Also show that

$$\mu_r' = E\left(\chi^r_{(n)}\right) = 2^{r/2} \frac{\Gamma[\frac{n+r}{2}]}{\Gamma[\frac{n}{2}]}$$

5. Let $X_1, X_2, \ldots, X_n$ is a random sample from N(μ, σ^2), find mean and variance of

$$S = \left\lfloor \sum_{i=1}^{n}(x_i - \bar{x})^2/(n-1) \right\rfloor^{1/2}$$

6. Let X_1 and X_2 be two independent normal variates each with the same mean μ and variance σ^2. Obtain the distribution of $Y = \frac{(X1+X2-2\mu)}{\sqrt{|X1-X2|^2}}$ and identify it.

7. Explain, stating clearly the assumptions involved, the t-test for testing the significance of the difference between two sample means.

8. If X is a χ^2 variate with n degrees of freedom, then prove that for large *n*, $\sqrt{2X} \sim N(\sqrt{2n}, 1)$.

9. A symmetric die is thrown 720 times. Use Chebychev's inequality to find the lower bound for the probability of getting 100 to 140 sixes.

10. For a random sample of size n from a continuous population whose p.d.f. f(x) is symmetric at x = μ, show that $f_r(\mu + x) = f_{n-r+1}(\mu - x)$,where $f_r(.)$ is the p.d.f. of $X_{(r)}$.
11. Define type I , type II errors and critical region with illustrations
12. Prove that if X has F - distribution with (m , n) d . f .and Y has the F- distribution with (n , m) d . f. , then for every a > 0 , $P(X \leq a) + P(Y \leq 1/a) = 1$.
13. If $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with mean μ_1 and variance σ_1^2 (finite) and $S_n = X_1 + X_2 + \ldots + X_n$, then show that , for $-\infty < a < b < \infty$,
$\lim_{n\to\infty} P[a \leq (S_n - n\mu_1)/\sigma_1\sqrt{n} \leq b] = \Phi(b) - \Phi(a)$, where $\Phi(.)$ is the distribution function of standard normal variate.

14. State and prove weak law of large numbers. If X_i can have only two values with equal probabilities i^{α} and $-i^{\alpha}$, then examine whether the weak law of large numbers can be applied to the independent variables $X_1, X_2, \ldots, X_n, \ldots$

15. Explain the terms 'standard error' and 'sampling distribution'. Show that in a series of n independent bernoulli trials with constant probability of success P the standard error of the proportion of successes is $\sqrt{PQ/n}$, where $Q = 1-P$

16. Given a random sample of size n from exponential distribution

$$f(x) = \alpha e^{-\alpha x}, \quad x \geq 0, \alpha > 0$$

show that $X_{(r)}$ and $W_{rs} = X_{(s)} - X_{(r)}$, $r < s$, are independent. Also find the distribution of $X_{(r+1)} - X_{(r)}$.

17. Show that for the rectangular distribution:

$$f(x) = \frac{1}{\theta 2}, \quad \theta_1 - \frac{1}{2}\theta_2 \leq x \leq \theta_1 + \frac{1}{2}\theta_2,$$

$$E\left(\frac{X(r) - \theta 1}{\theta 2}\right) = \frac{r}{n+1} - \frac{1}{2}.$$

18. Define null and alternative hypotheses. Discuss the test of significance for single proportion for large samples .Also obtain $100(1-\alpha)\%$ confidence interval for single proportion.

19. Let $\{X_n\}$ be a sequence of independent random variables with probability distribution

$$P(X_k = 0) = 1 - k^{-2\alpha}, \qquad P(X_k = \pm k^{\alpha}) = 1/2.\, k^{-2\alpha},$$

Examine whether the central limit theorem holds good for the sequence $\{X_n\}$.

20. Define convergence with probability one and convergence in probability and establish the relation between them.

21. For the t-distribution with n d. f., prove that $\mu_{2r} = \frac{n(2r-1)}{(n-2r)}\mu_{2r-2}$, $n > 2r$. Hence find β_1 and β_2.

22. What is contingency table? Describe how the χ^2 distribution may be used to test whether the two attributes in an m x n contingency table are independent.

23. Let $x_1, x_2, \ldots, x_n$ be independent observations from a normal universe with mean μ and variance σ^2 and let $\bar{x}$ and s^2 the sample mean and sum of the squares of the deviations from the mean respectively. Let x_{n+1} be one more observation independent of previous ones. Obtain the sampling distribution of

$$U = \frac{(x_{n+1} - \bar{x})}{s} \cdot \sqrt{\frac{n(n-1)}{(n+1)}} .$$

24. If $X_1, X_2, \ldots, X_n$ is a random sample from a $N(\mu, \sigma^2)$, find the mean and variance of

$$. \ S = \left(\frac{1}{n-1} \sum_{i=1}^{n} \left(X_i - \bar{X} \right)^2 \right)^{\frac{1}{2}}$$

25. If $X \sim F_{(m,n)}$, *then* obtain the distribution of $U = \frac{mX}{n+mX}$.

STAT C-302 SURVEY SAMPLING AND OFFICIAL INDIAN STATISTICS

1. Discuss briefly the main steps involved in a sample survey. Discuss the basic principles of sample surveys. What are the advantages of sample surveys over complete enumeration?
2. Define simple random sampling without replacement from a finite population. Derive the unbiased estimator of the population mean and find its sampling variance.
3. Define simple random sampling without replacement from a finite population. Derive the unbiased estimator of the population mean and find its sampling variance.
4. For srswor, prove that

$$cov(x_i\, ,\ \bar{y}_n) = \frac{N-n}{Nn}\cdot\frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X}_N)(Y_i - \bar{Y}_N) = \frac{N-n}{n(N-1)}\ cov(X,Y).$$

Also evaluate $E(\bar{x}_n\bar{y}_n)$.

5. Describe the method of determining the sample size in case of simple random sampling so as to meet the desired margin of error and confidence coefficient, stating the assumptions made.

6. From a simple random sample of size n drawn from N units by srswor, a simple random sub-sample of n_1 units is duplicated and added to the original sample. Show that the mean based on $(n + n_1)$ units is an unbiased estimator of the population mean. Also obtain its variance. How does it compare with the variance of the estimator based on n units only?

7. A sample of size n is drawn from a population having N units by simple random sampling without replacement. A sub-sample of n_1 units is drawn from the n units by simple random sampling without replacement. Let $\bar{y}_1$ denote the mean based on n_1 units and $\bar{y}_2$ be the mean based on $(n - n_1)$ units.

Consider the estimator of the population mean $\bar{Y}_N$ given by:

$$\bar{y}_w = w\bar{y}_1 + (1-w)\bar{y}_2.$$

(i) Show that $E(\bar{y}_w) = \bar{Y}_N$, and obtain its variance.
(ii) Find the optimal value of w for which $V(\bar{y}_w)$ is minimum.
(iii) Find the optimal estimator and its variance.

8. A simple random sample of size $n = n_1 + n_2$ with mean $\bar{y}_n$ is drawn from a finite population of size N by srswor. Further, a simple random sub-sample of size n_1 is drawn from it by srswor with mean $\bar{y}_1$, show that

$$(i)\ v(\bar{y}_1 - \bar{y}_2) = \left(\frac{1}{n_1} + \frac{1}{n_2}\right)S^2$$

$$(ii)\ v(\bar{y}_1 - \bar{y}) = \left(\frac{1}{n_1} - \frac{1}{n}\right)S^2$$

$$(iii)\ cov(\bar{y}\,, \bar{y}_1 - \bar{y}_2) = 0$$

where, S^2 is the population mean square and $\bar{y}_2$ is the mean of remaining $(n - n_1)$ units.

9. In SRSWOR, prove that sample mean square is an unbiased estimator of population mean square.
10. Justify the following statements:
    (i) The smaller the size of stratum, the smaller should be the size of sample to be selected there from .
    (ii) The smaller the variability within a stratum, the smaller should be the size of sample selected from the stratum.
    (iii) The cheaper the cost per unit in a stratum, the larger should be the size of sample selected from that stratum.

    Hence obtain minimum size required for estimating population mean with fixed variance fixed cost under optimum allocation.
11. (a) Define proportional and Neyman allocations. Compare stratified random sampling under

Neyman and proportional allocation with simple random sampling without replacement.

(b) Compare simple random sampling without replacement with stratified random sampling under arbitrary allocation.

12. Obtain the estimated gain in precision due to arbitrary stratification over simple random sampling without replacement for estimating the populations mean $\bar{Y}_N$.

13. With two strata, a sampler would like to have $n_1 = n_2$ for administrative convenience, instead of using the values given by the Neyman allocation . If $V(\bar{y}_{st})$ and $V(\bar{y}_{st})_{opt}$ denote the variances given by the $n_1 = n_2$ and the Neyman allocations respectively , show that the fractional increase in variance is

$$\frac{V(\overline{y_{st}})-V(\overline{y_{st}})opt}{V(\overline{y_{st}})opt} = \left(\frac{r-1}{r+1}\right)^2$$

14. Discuss briefly the practical difficulties in adopting Neyman optimum allocation. Also obtain the initial sample size when modified Neyman optimum allocation is better than proportional allocation.

15. Obtain an estimate of the gain in precision due to stratification under proportional allocation relative to simple random sampling for estimating the population mean $\bar{Y}_N$.

16 If there are two strata and if φ is the ratio of actual $\frac{n_1}{n_2}$ to the Neyman optimum allocation $\frac{n_1}{n_2}$, show that whatever be the values of N_1, N_2, S_1 and S_2, the ratio $\frac{V(\bar{y}_{st})_{min}}{V(\bar{y}_{st})}$ is never less than $4\varphi(1+\varphi)^{-2}$ where f.p.c. are negligible.

17. State the practical difficulties in adopting Neyman method of allocation of a sample to different strata. How much would be the increase in variance, on the average, if the allocation is based on the estimates of strata mean squares? Also, compare it with proportional allocation.

18. Explain the concept of post stratification. Show that, for large samples, the post stratification is as precise as stratified sampling with proportional allocation.

19. In usual notation, prove that (The notations have their usual meaning.)

$$V\left(\bar{y}_{sys}\right)=\frac{k-1}{nk}S^2_{wst}\left[1+\left(n-1\right)\rho_{wst}\right]$$

20. (a) Define linear systematic and circular systematic sampling. Prove that in systematic sampling positive intra class correlation coefficient between units of same systematic sample inflates the

variance of the systematic sample mean.

(b) Derive the variance of systematic sampling in terms of intra-class correlation Coefficient ρ. Prove that reduction in this variance over srswor will be 100% if ρ assumes the minimum possible value. If ρ assumes the maximum value, X

21. What is the relative efficiency of systematic sampling over simple random sampling?

22. Define linear systematic and circular systematic sampling. Prove that systematic sampling is more precise than srswor if the variation within the systematic samples is larger than population variation as a whole.

23.(a) Show that a systematic sample has the same precision as the corresponding stratified random sample with one unit per stratum if $\rho_{wst} = 0$, the notation has its usual meaning.

(b) Derive the condition under which a systematic sample has the same precision as the corresponding stratified random sample with one unit per stratum.

24. Carry out a comparison of simple random sampling, stratified random sampling and systematic sampling in the presence of a linear trend in the population

25. Explain the Yates' end corrections in systematic sampling for a population with a linear trend and hence show that the sample mean obtained after applying the end corrections overlaps the population mean

26. Prove that, in the presence of a linear trend, the variance of a stratified sample is only $\frac{1}{n}th$ of the variance of a systematic sample and the latter is also approximately $\frac{1}{n}th$ the variance of a random sample. Hence show that the systematic sample with Yates' end correction provides the exact population mean.

27. Prove that the systematic sampling will be more efficient than SRSWOR if the intra class correlation coefficient $\rho < -\frac{1}{N-1}$.

28. For a population with linear trend, Show that the notations have their usual meaning,

$$V\left(\bar{y}_{st}\right) \le V\left(\bar{y}_{sys}\right) \le V\left(\bar{y}_{n}\right)_{SRS}.$$

Also, show that the weighted sample mean in systematic sampling with Yates' end corrections provides the exact population mean.

29 . For ratio estimator derive, to the first approximation, its bias and find the condition under

which this bias vanishes altogether. Also obtain the variance of the ratio estimator.

30.(a) Prove that the necessary and sufficient condition for ratio estimator to be unbiased when results are calculated to the first approximation, is that regression of y on x is linear and the line passes through the origin.

(b) If the coefficients of variation of y and x are equal then show that,under first approximation, relative variance of ratio estimator R_n is twice the relative bias of it.

31. Derive the linear regression estimator from the difference estimator. Derive, to the first approximation, the expressions for the bias and mean square of the linear regression estimator.

32. Define Difference estimator and derive from it the regression estimator. Also obtain the variance of regression estimator under first approximation.

33. If the coefficient of variations of y and x are equal, then prove that, under first approximation, (The notations have their usual meaning).

$$MSE_1\left(\frac{R_n}{R_N}\right)=2\left(\frac{1}{n}-\frac{1}{N}\right)C^2(1-\rho)=2Bias_1\left(\frac{R_n}{R_N}\right).$$

34. Compare the ratio estimator with simple random sampling and derive the condition under which they are equally efficient.

35 Compare the regression estimator with the ratio estimator and the simple random Sample mean, assuming the formulae for the variances of these estimators.

36. If y and x are unbiased estimators of the population totals of Y and X respectively, Show that the variance of ratio estimate $\frac{y}{x}$ can be approximated by $c_y^2 - c_x^2$, where c_x and c_y are coefficient of variation of x and y respectively. (The correlation Coefficient between $\frac{y}{x}$ and x is assumed to be negligible).

37. Prove that the relative efficiency of cluster sampling as compared to simple random sampling without replacement increases with the increase of mean squares within clusters. Also, obtain an estimate of this relative efficiency.

38. Calculate the efficiency of cluster sampling relative to simple random sampling in terms of the intra class correlation coefficient for negligible sampling fraction .

39. (a) Establish the results which justify the following statements:

(i) Efficiency of cluster sampling increases as mean squares within clusters increases.
(ii) If the clusters are formed of random samples of elements of population, they will on an average, be as efficient as the individual elements themselves.

(b) Prove that if clusters are formed at random, cluster sampling is as efficient as simple random sampling without replacement.

40. Prove that the mean of cluster means $\bar{\bar{y}}$ is an unbiased estimator of population mean with variance given as

$$V(\bar{\bar{y}}) = \frac{N-n}{N-1} \cdot \frac{\sigma^2}{nM}[1 + (M-1)\rho]$$

41. Find the variance of an estimator of population mean based on cluster sampling in terms of intra-class correlation coefficient between the elements of a cluster. Hence, prove that the increase in the size of the cluster usually leads to a substantial increase in the sampling variance.

42 (a) Prove that efficiency of cluster sampling relative to simple random sampling without replacement increases with the increase of mean squares within clusters. Also obtain an estimate of the relative efficiency.

(b) Obtain the estimated relative efficiency of cluster sampling with respect to srswor.

43. Define two-stage sampling with example. In two-stage sampling with equal first stage units prove that the sample mean is an unbiased estimator of population mean.

44. What is meant by non-response in sample surveys? Explain the Hansen and Hurwitz technique for removing the bias arising from non-response in mail surveys. Obtain the variance of the estimator of population mean and minimize it for the fixed cost.

45. What is meant by non-response in sample surveys? What is the effect of non-response on the estimates? Prove the statement "the estimate obtained from incomplete samples may be reasonable if response and non-response classes are alike".

46. Write note on the following:

Sampling Frame, Sampling Units, Sampling and non sampling error, Sampling fraction, Finite population correction factor, principles of Validity and optimization,

47. What is primary and secondary data ? Mention methods each of collecting primary and Secondary data with example.

48. Discuss briefly the present statistical system in India,

49. Discuss briefly about the statistical wing, MOS & PI and its functions.

50. Describe briefly the role and responsibilities of CSO and NSSO. Name their main divisions and Publications.

51. What is the background and objectives of COCSSO?

52. Write about National Statistical Commission in India mentioning its two important functions.

53. (a) Name four Government of India's principle publications each on population and industry.

(b) Name three Government of India's NAD publications related to the National Income estimates.

54. Discuss briefly about the use of information technology in the Indian Statistical system.

55. Write briefly on economic census and follow up surveys

56. Discuss the main functions of statistical system and office of the Registrar General of India .

57. Why Statistical Audit is necessary? Mention five audited items.

58. Discuss the functions of Directorate of Economics and Statistics.

59. What are the sources and methods of collection of data? Also what do you mean by editing of data.

60. Write notes on the following:

(i) National Statistical Commission in India.

(ii) Potential of the NSS Survey mechanism and data.

(iii) The States' Statistical systems

(iv) Economic Census

(v) Objectives of NSSO

(vi) Functions of CSO

(vii) Determinants of quality of statistics.

61. Mention important recommendations each relating to the functioning of NSSO by the Commission regarding (i) organizational aspects & (ii) survey programme.

STAT C-303
MATHEMATICAL ANALYSIS

Question based on Real Analysis

Short type

1. (a) Write down the Supremum and Infimum of the following

$$S = \left\{1 - \frac{(-1)^n}{n} \ : \ n \in N\right\}$$

(b) Give an example of a set which is

(i) Infinite and bounded.

(ii) Neither bounded above nor bounded below.

(c) Write down the set of all limit points of

(i) set Z of integers

(ii) set of rational numbers.

(d) (i) Give an example of a bounded sequence which is not convergent.

(ii) If $\lim\limits_{n\to\infty} a_n = l$ and if $< S_n >$ be the sequence defined as

$$S_n = {(a_1 + a_2 + \cdots + a_n)}/{n} \quad \text{then find} \quad \lim_{n\to\infty} S_n$$

(e) Show that the series $1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \cdots + \left(\frac{2}{3}\right)^{n-1}$ converges. (2)

(f) What is an alternating series? Give example. State Leibniz test for alternating series.

(g) Examine for continuity at x = 0 for the function defined as

$$f(x) = \begin{cases} \frac{Sin2x}{Sinx} & , x \neq 0 \\ 1 & , \ x = 0 \end{cases}$$

(h) Give the geometrical interpretation of Rolle's Theorem.

(i) For what values of '**a**' does $\frac{Sin2x + a\ Sinx}{x^3}$ tend to a finite limit as x→0 and find this limit.

Lengthy question

2. (a) Let S be a non empty set of real numbers bounded below. Then prove that a real number t is the infimum of S iff the following conditions hold

(i) $x \geq t, \quad \forall\, x \in S$

(ii) for each positive real number ε, there is a real number $x \in S$ such that $x < t + \varepsilon$

(iii) (b) If M and N are neighbourhoods of a point p, then prove that M ∩ N is also a neighbourhood of p

3. (a) Define

(i) An Open set

(ii) A Monotonic sequence

(iii) A Bounded set

Show that the intersection H of an arbitrary family ζ of closed sets is a closed set.

(b) Show that a set is closed iff it contains all its limit points.

(a) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ exists and lies between 2 and 3.

(b) Show that every Cauchy sequence is bounded.

1. (a) Show that the series $\sum_{n=1}^{\infty} 1/_{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

(b) Test for convergence of the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n \quad \text{for positive values of x}$$

2. (a) Define Conditionally Convergent Series. Show that the series

$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ is Conditionally Convergent.

(b) Let f be a function defined on [0,1] as

$$f(x) = \frac{1}{2^n}, \text{when } \frac{1}{2^{n+1}} \leq x \leq \frac{1}{2^n}, \quad n = 0,1,2,3,\ldots$$

$$f(0) = 0$$

Show that f is continuous except at the points $x = \frac{1}{2}, \frac{1}{2^2}, \ldots, \frac{1}{2^n}, \ldots$. Describe the nature of discontinuity at each of these points.

3. (a) Prove that for any quadratic function $px^2 + qx + r$, the value of θ in Lagrange's Mean Value Theorem is always ½ , whatever p,q,r,a,h may be.

(b) Obtain Maclaurin's series expansion of sinx.

(c) Evaluate any **two** of the following

(i) $\lim_{x\to 0}(1+x)^{1/x}$

(ii) $\lim_{x\to 0}\left(x^k \log x\right)$ for each fixed $k \in R^+$

(iii) $\lim_{x\to 0}(\tan x)^{\sin 2x}$

Short type

1. (a) Write down the Supremum and Infimum of the following

$$S = \left\{1 - \frac{1}{n} \ , n \in N\right\}$$

(b) Give an example of a set which

(i) has infinite limit points.

(ii) is closed but not an interval.

(c) Write down the set of limit points of

(i) S = {1,2,3,4}.

(ii) set of real numbers.

(d) (i) Give an example of a bounded sequence which is not a Cauchy sequence.

(ii) Let $< a_n >$ be a sequence such that $a_n > 0$ for all n. If $\lim_{n\to\infty} a_n = l > 0$ and $P = (a_1 a_2 \dots a_n)^{1/n}$. Then find $\lim_{n\to\infty} P_n$

(2)

(e) Show that the series 1+2+3+…+n+…diverges.

(f) Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ converges.

(g) Examine for continuity at x=0 for the function given as

$$f(x) = \begin{cases} \frac{Sin3x}{x} & , x \neq 0 \\ 1 & , \ x = 0 \end{cases}$$

(h) Give the geometrical interpretation of Lagrange's Mean Value Theorem.

(i) Find $\lim_{x\to 0+} (x)^x$

Lengthy question

2. (a) Let S be a non empty set of real numbers bounded above. Then prove that a real number s is the supremum of S iff the following conditions hold

(i) $x \leq s, \ \forall x \in S$

(ii) for each positive real number ε, there is a real number $x \in S$ such that $x > s - \varepsilon$

(b) If x and y be any positive real numbers, then prove that there exists a positive integer n such that ny > x

3. (a) Define

(i) A Bounded above set

(ii) An Open set

(iii) Monotonically increasing Sequence.

Prove that the intersection of two open sets is an open set.

(b) Prove that the derived set of any set is a closed set.

4. (a) Show that the sequence $<a_n>$ defined by

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n} \quad n \in N. \quad \text{converges}$$

(b) Show that every bounded monotonically increasing sequence converges.

5 (a) Show that the series $\sum_{n=1}^{\infty} {}^{1}/_{n^p}$ converges if $p > 1$ and diverges if $p \le 1$.

(b) Test for convergence the series

$$\sum_{n=1}^{\infty} \frac{(n-1)}{\sqrt{n^3+1}} x^n \quad \text{for positive values of x}$$

6. (a) Show that every absolutely convergent series is convergent.

(b) Let f be a function defined on [0,1] as

$$f(x) = \frac{1}{2^n}, \text{when } \frac{1}{2^{n+1}} \le x \le \frac{1}{2^n}, \quad n = 0,1,2,3, \ldots$$

$$f(0) = 0$$

Show that f is continuous except at the points $x = \frac{1}{2}, \frac{1}{2^2}, \ldots, \frac{1}{2^n}, \ldots$. Describe the nature of discontinuity at each of these points. (6)

7. (a) Show that there is no real number k for which the equation $x^3 - 3x + k = 0$ has two distinct roots in [0,1].

(b) Obtain Maclaurin series expansion of cosx.

(c) Evaluate any **two** of the following

(i) $\lim_{x \to 0} (1 + x)^{1/x}$

(ii) $\lim_{x \to 0} \left(\frac{1}{e^x - 1} - \frac{1}{x} \right)$

(iv) $\lim_{x \to 0} (\cot x)^{\sin 2x}$

Set 3 (lengthy)

1. (a) State order completeness property of real numbers.Prove that the set of rational numbers is not order complete.

   (b) Prove that the intersection of arbitrary family of open set need not be open.

2. (a) Prove that every infinite bounded set of R has at least one limit point.

   (b) Define neighbourhood of a point.Show that, if M and N are neighbourhoods of a point p, $M \cap N$ is also neighbourhood of p.

3. (a) State and prove Cauchy's first Theorem on limits.

   (b) Prove that the sequence $\langle a_n \rangle$ defined by

   $$a_1 = \sqrt{7}, \quad a_{n+1} = \sqrt{7 + a_n}$$

   converge to the positive root of the equation $x^2 - x - 7 = 0$.

4 (a) State and prove Monotone Convergence theorem.

(b) Let $\langle a_n \rangle$ be a sequence such that $a_n \neq 0$ for any n and $\frac{a_{n+1}}{a_n} \to l$.

Prove that if $|l| < 1, \quad a_n \to 0.$

(c) Show that the series $1 + r + r^2 + ...$ converges if 0<r<1 and diverges if

$r \geq 1.$

5 (a) Test for the convergence the series

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$$

(b) State and prove Gauss test for convergence of an infinite series of positive

terms

NUMERICAL ANALYSIS

Q1 Find the third divided difference with arguments 2 , 4,9,10 of the function $f(x) = x^3 - 2x.$

Q2 Find the relation between α, β, γ in order that $\alpha + \beta x + \gamma x^2$ may be expressible in one term in the factorial notation.

Q3 If D, E, Δ and ∇ are the operators with usual meaning and if $hD = U$, where h is the interval of differencing, prove that :

(i) $\nabla^2 = U^2 - U^3 + \frac{7}{12}U^4 - \dots\dots\dots\dots\dots$

(ii) $U^2 = \nabla^2 + \nabla^3 + \frac{11}{12}\nabla^4 + \dots\dots\dots\dots\dots\dots$

Q4 Prove that

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \cdots = e^x \left[u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + \cdots \right]$$

Q5) Show that

$$u_{2n} - \binom{n}{1}2^1 u_{2n-1} + \binom{n}{2}2^2 u_{2n-2} \dots\dots\dots + (-2)^n u_n = (-1)^n (c - 2an)$$

Where $u_x = ax^2 + bx + c$

Q6) If u_x be a function whose differences when the increment of x is unity are denoted by $\delta u_x, \delta^2 u_x, \dots\dots$ and $\Delta u_x, \Delta^2 u_x, \dots\dots$ when the increment of x is n, then if $\delta^2 u_x, \delta^2 u_{x+1}, \delta^2 u_{x+2}$ .... are in G.P with common ration q, show that

$$\frac{\Delta u_x - n\delta u_x}{(q^n - 1) - n(q - 1)} = \frac{\delta^2 u_x}{(q - 1)^2}$$

Q7) State Newton's divided difference formula. Hence obtain Newton Gregory forward difference interpolation formula.

Q8) Show that for the interpolation of $f(x)$ relative to $0, \alpha, 1$ Lagrange's formula gives:

$$f(x) \cong \left[1 - \frac{x(x-\alpha)}{1-\alpha}\right] f(0) + \frac{x(1-x)}{1-\alpha} \times \frac{f(\alpha) - f(0)}{\alpha} + \frac{x(x-\alpha)}{1-\alpha} \times f(1)$$

Also show that if $\alpha \to 0$, it reduces to :

$$f(x) = (1 - x^2)\, f(0) + \; x(1 - x)f'(0) + x^2 f(1)$$

Q9) By means of usual notations $\Delta, \nabla,$ and δ, establish the following relation

$$\Delta^r f_i = \delta^r f_{i+\frac{r}{2}} = \nabla^r f_{i+r} = r!\, h^r f\left[x_{i,} x_{i+1}, - - - -, x_{i+r}\right], \text{ where } f_i = f(x_i)$$

Q10) The value of $f(x)$ are given at a, b and c. Show that under certain conditions to be mentioned, the maximum and minimum is attained at

$$x = \frac{\sum (b^2 - c^2)\, f(a)}{[2 \sum (b - c) f(a)]}$$

Q11) Using Newton-Cotes integration formula, derive Simpson's $\frac{1}{3}$rd rule

Q12) If $f(x)$ is a function whose fifth differences are constant, then $\int_{-1}^{1} f(x)dx$ can be expressed in the form $pf(-\alpha) + qf(0) + pf(\alpha)$.

Find the values of p, q and α.

Q 13 Prove that

$$u_1 + u_2 + \; \ldots \ldots + u_n = \binom{n}{1} u_1 + \binom{n}{2} \Delta u_1 + \binom{n}{3} \Delta^2 u_1 + \cdots + \Delta^{n-1} u_1$$

Q14) Prove that

$$u_0 + u_1 + u_2 + \cdots + u_n = \binom{n+1}{1} u_0 + \binom{n+1}{2} \Delta u_0 + \; \ldots + \binom{n+1}{n+1} \Delta^n u_0$$

Q 15) Estimate the missing term in the following table:

x:	1	2	3	4	5	6	7
y:	2	4	8	-	32	64	128

Explain why the result differ from 2^4

Q16) Derive Lagrange's interpolation formula and show that the sum of Lagrangian coefficients is unity.

Q17) By means of Lagrange's interpolation formula, prove that approximately

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right]$$

Q 18) Obtain Gauss forward central difference interpolation formula for equal intervals.

Q19) State Newton- Cotes integration formula and hence derive Simpson's $\frac{1}{3}$rd rule and Simpson's $\frac{3}{8}$th rule.

Q20) Calculate by Trapezoidal rule and Simpson's $\frac{1}{3}$rd rule, an approximate value of $\int_{-3}^{3} x^4 dx$ by taking seven equidistant ordinates. Compare it with the exact value.

Q21) If *p*, *q*, *r* and *s* be the successive entries corresponding to equidistant arguments in a table, show that when third differences are taken into account, the entry corresponding to the argument half-way between the arguments of q and r is $A + \frac{B}{24}$, where A is the arithmetic mean of q and r and B is the arithmetic mean of 3*q*-2*p*-*s* and 3*r*-2*s*-*p*

Q22) Show that for interpolation of $f(x)$ relative to 0, α, 1 Lagrange's formula gives:

$$f(x) \cong \left[1 - \frac{x(x-\alpha)}{1-\alpha}\right] f(0) + \frac{x(1-x)}{1-\alpha} \times \frac{f(\alpha) - f(0)}{\alpha} + \frac{x(x-\alpha)}{1-\alpha} \times f(1)$$

Also show that if $\alpha \to 0$, it reduces to

$$f(x) = (1 - x^2)f(0) + x(1-x)f'(0) + x^2 f(1).$$

Q 23) State and prove Euler Maclaurin summation formula .

Q24) If $f(x)$ is a polynomial in *x* of third degree and

$u_{-1} = \int_{-3}^{-1} f(x)dx,\ u_0 = \int_{-1}^{1} f(x)dx, u_1 = \int_{1}^{3} f(x)dx,$ then

$$f(0) = \frac{1}{2}\left(u_0 - \frac{\Delta^2 u_{-1}}{24}\right)$$

Q 25) If the fourth differences of $f(x)$ can be neglected and

$$\int_{-1}^{1} f(x)dx = \frac{2}{3}[f(x_1) + f(x_2) + f(x_3)]$$

Find the values of x_1, x_2 and x_3

Q 27) Derive Stirling interpolation formula .

Q 28) Solve the following difference equations:

(i) $u_{x+2} - 2mu_{x+1} + (m^2 + n^2)u_x = m^x$

(ii) $u_{x+1} - au_x = Sin\ bx$

(iii) $u_{x+2} - 7u_{x+1} - 8u_x = x^{(2)}.2^x$

(iv) $u_{x+1} - 2u_x^2 + 1 = 0$

(v) $u_{x+2} - 7u_{x+1} + 10u_x = 12.5^x$

(vi) $u_{x+2} - 7u_{x+1} - 8u_x = x^{(2)}\ 2^x$

(vii) $u_{x+1} - 2u_x^2 + 1 = 0$

(viii) $u_{x+4} + u_x = 0$

(ix) $u_{x+2} - 4u_x = 9x^2$

(x) $u_{x+1}u_x - a^x(u_{x+1} - u_x) + 1 = 0$

(xi) $u_{x+1} = 2u_x\sqrt{1 - {u_x}^2}$

SKILL ENHANCEMENT ELECTIVE
STATISTICAL DATA ANALYSIS USING R

Statistical Data Analysis Using R (SEE-2)

1. Fill in the blanks:
   a. A command used for "logarithm of x with base n" is _____________.
   b. R code used to append an observation to a vector L is given by _________.
   c. Horizontal line can be drawn using a command _______ (h = value).
   d. Spline command is used in R for drawing a ___________ curve.
   e. CRAN in R stands for Comprehensive R _________.
   f. A command used to extract 6th element from a vector x of 8 elements is ______.
   g. For a given vector x = c(3, 1, 2, 5, 4, 8, 9, 5), the values obtained by using cummax(x) are ________________.
   h. A command/R code abline (v = value) is used for drawing __________ line.
   i. Graphical window can be divided into several parts using the graphical instruction ____________________.
2. The following set of R codes

   length(marks) = 5

   marks

   for a vector marks = c(9, 8, 7, 10) produce the output as __________.
3. Can we use customized x –axis limits and y – axis limits in a graphical representation. Give example.
4. Write R codes to obtain P(X≤3),where X~Binomial(n=12,prob.=0.4).
5. Write the output of the following R Codes:

   X <- seq(10,70,20)

   X
6. Write the arguments used in graphical representation of R for the line type and line width.
7. How do you import data in R from excel? What should be the file type of excel sheet saved?
8. What are the difference in high level and low level plots and name one each of high level and low level plot.
9. If x is a vector draw a histogram for a grouped frequency distribution with equal class intervals.
10. For a given vector, draw a pie chart with the initial angle 90 degrees and it is in clockwise direction.
11. Draw frequency polygon curve for the raw data of length n.
12. Discuss d, r, p, q functions of a family of distribution with respect to uniform distribution.
13. A unbiased die is rolled five times. Write a R-command to get the probability of three or more rolls of four.
14. What are the five basic classes of objects in R? Give examples to show how values are assigned to each class of variable.
15. Write R commands to calculate $\frac{1}{n}\sum_1^n |x_i - \bar{x}|$
16. Given the frequency distribution $x_i | f_i$, draw less than and more than ogives in a single plot.
17. What is the significance of having a plotting character (pch) in R.
18. Write P(X=x)for x=0,1,2,3,4,5 when lambda = 1 and where X~Poisson(lambda).

19. Write the use of summary and table function used in R.
20. Write the output of the following R Codes:

```
p <- c(32,54,38,44)
x <- c(10,20,30,40,50,60,70)
y <- cut(p, breaks= x)
```

21. Draw a histogram for a grouped frequency distribution with unequal class intervals.
22. Given the frequency distribution, $x_i|f_i, (i = 1,2, \dots ,6)$ draw less than and more than ogives in two different plots.
23. Draw frequency polygon curve for a data given as a vector r.
24. For a given vector, draw a pie chart with the initial angle 60 degrees and it is in anti-clockwise direction.
25. Explain Paired t-test for difference of means. Also interpret the results as obtained in R. Write R codes for mean, variance, median and mode for both the samples used in the above t-test.
26. Write a R-code to fit a binomial distribution for given $x_i|f_i, (i = 1,2, \dots ,6)$ and also test the goodness of fit.
27. For the given vectors x and y, fit a line of regression and plot a graph.
28. For a given raw data, obtain the grouped frequency data with 6 class intervals. Also obtain the mid value for each class and the cumulative frequencies.
29. Fit a poisson distribution for given $x_i|f_i, (i = 1,2, \dots ,6)$ and also test the goodness of fit.
30. Explain t-test for difference of means when the samples are drawn from same population. Also interpret the results as obtained in R. Write R codes for mean, variance, median and mode for both the samples used in the above t-test.